

Name of the Course : **B.Sc. (H) Mathematics (CBCS)**
 Unique Paper Code : **32357505_OC**
 Name of the Paper : **DSE- Discrete Mathematics**
 Semester : **V Semester**
 Duration : **3 hours**
 Maximum Marks : **75 Marks**

Instructions for Candidates:

Attempt any four questions. All questions carry equal marks.

1. Let (P_1, \leq) be ordered set of all divisors of 5 and (P_2, \leq) be ordered set of all divisors of 9, both ordered with divisibility. Draw the diagram of direct product $P_1 \times P_2$. Is it same as the diagram of the lattice of factors of 20, justify, also Prove or Disprove $P_1 \times P_2$ is a lattice. Further, prove the result that a lattice (L, \leq) is a chain if and only if every nonempty subset of L is a sublattice of L .

2. Use the Karnaugh map method to simplify the polynomial p given by

$$p = (x' + y)(x' + z) + xyz$$

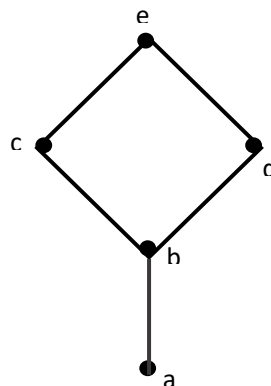
Does the expression $x'y'$ imply the expression $xy'z' + x'y'z + x'y'z'$. Give reason for your answer

Give the symbolic representation of the circuit q given by

$$q = xy'z((x' + z) + (y + z'))$$

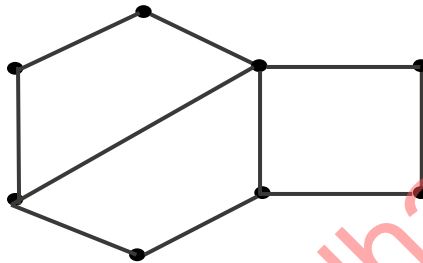
Also, draw the contact diagram of above circuit q .

3. Let $A = \{a, b, c, d, e\}$ be a partial ordered set with the given Hasse diagram

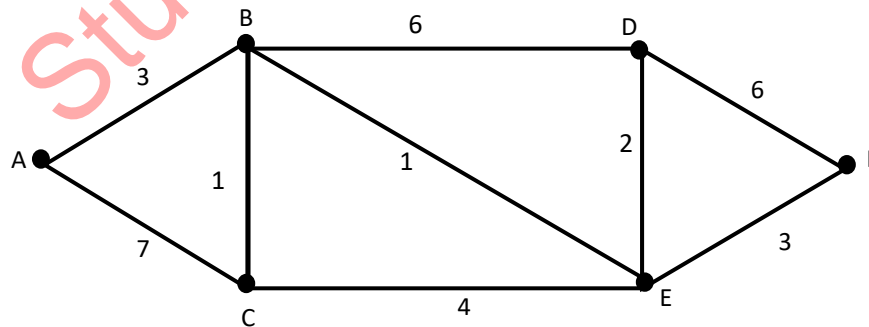


Find the meet (\wedge) and join (\vee) of subsets of $\{a,c\}$ and $\{b,e\}$. Let $B = \{1, 2, 3, 4, 5\}$ be order isomorphic to A under the following order isomorphism: $f(a) = 1, f(b) = 3, f(c) = 5, f(d) = 2, f(e) = 4$. Draw the Hasse diagram of poset B . Also prove that in a lattice every non empty subset has a supremum and infimum.

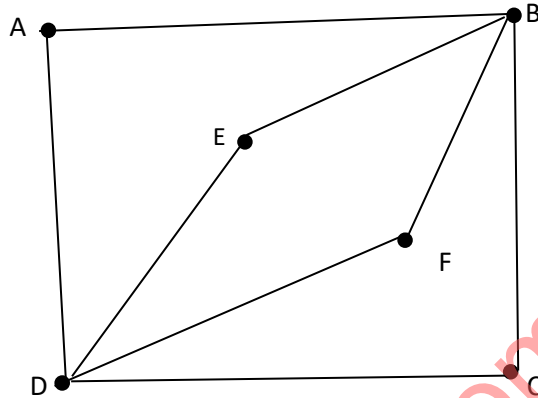
4. Draw the graph $K_{4,4}$. How many edges does this graph has? Also, find the adjacency matrix of this graph. Does there exist a graph with 32 edges and 12 vertices, each of degree 4 or 6? Justify your answer. For the graph given below, Either give the bipartition sets or explain why the graph is not bipartite.



5. Use the original form of Dijkstra's Algorithm to find shortest path from A to F in the graph given below. Label all vertices. Explain algorithm.



Consider the following Graph. Check whether the graph is Eulerian. Explain your Answer.



6. Find elements a, b, c in N_5 which do not satisfy the following law:

$$(a \vee b = c \vee b \text{ and } a \wedge b = c \wedge b) \Rightarrow a = c$$

Deduce that a lattice L is distributive if and only if the above law holds for all $a, b, c \in L$. Verify whether the lattice L in the figure given below is modular and /or distributive, by using M_3-N_5 theorem.

